On the Theory of Aberration. By H. C. Plummer, M.A.

1. The appearance of Professor Turner's paper on Aberration, in which he has placed in a clearer light the geometrical implications of the accepted law of aberration, suggests that it may be not inopportune to examine, even superficially, the physical aspects of the same question. It is well known to astronomers that recent determinations of the constant of aberration, executed with unimpeachable skill and care, have yielded values which are greater than can be reconciled with the known values of the solar parallax and the velocity of light. Of the three related constants it remains the most uncertain, and the discrepancy suggests either that the simple theory of aberration is not entirely correct, or that the direct determinations of the constant are affected by some source of systematic error which has hitherto been overlooked. The problem is clearly one of great importance, and a review of the position may be useful if it only serves to define the obscurity. The discrepancy is of course very small, yet it is scarcely to be dismissed as a mere result of the accidental errors of observation.

The question will be regarded here from the general standpoint of the undulatory theory, and this rather because it provides as simple a view as any other than because it is to be considered essential. Those text-books may well seem open to criticism in which it is sought to *explain* aberration by simple considerations based on the corpuscular theory of light. The proper view, it may be thought, is to regard the law of aberration as an inductive result of observation which must be taken into account by the physicist in formulating a theory of light.

A different view is, however, possible, and further consideration tends to lead us in this direction. According to the alternative view, the physical importance of astronomical aberration is chiefly historical, and its value in the construction of hypotheses is much less than we might at first suppose.

2. Let us begin by considering the simplest case—that of maximum aberration *in vacuo*. At the time t let fc (fig. 1) be the position of the axis of collimation of a telescope, being defined by a fixed wire at the focus f and the principal point c, and let acb be a plane at right angles to fc. The telescope moves in a direction at right angles to itself, and the axis arrives at the position FC at the time T. Suppose that S is a star, the light from which arrives in a plane wave w, and is converted by the lens system into a spherical wave s. If the light of the star reaches c at the time t and w is parallel to acb, the light will travel from c to the focus f in free ether. Let U be the velocity of light, V the velocity of the telescope, and f the focal length. The passage of the light from c to f will occupy a time $f/\mathrm{U}=\mathrm{T}-t$. In this time the telescope will have moved a distance $f\mathrm{F}=f\mathrm{V}/\mathrm{U}$. Hence the angular displacement of the observed image will be $f\mathrm{CF}=\tan^{-1}\mathrm{V}/\mathrm{U}$. It is assumed here that the lens produces a flat field. If now a bright micrometer wire be

made to bisect the image at f, both the image and the wire will act as coincident sources of light, and no further aberration will occur in the passage through the eyepiece to the retina of the observer.

In order to arrive at this result, which is in accordance with observation, we have made three assumptions: (1) that the light arrives in a plane wave at c, the principal point corresponding to f; (2) that on arriving there the plane wave is instantaneously converted into a spherical wave of radius f; (3) that the wave travels thence to the focus in undisturbed ether. Separately these assumptions are certainly not true, but together they may approximate to the truth. If the lens can fairly be considered as a thin lens in proportion to the focal length, they will not be

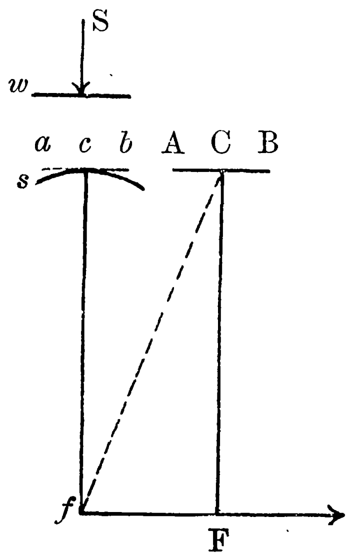


FIG. 1.

greatly in error. But at first sight we have here the possibility of a disturbing factor in the dimensions of the lens system.

For the sake of clearness and simplicity we have considered the case in which the telescope is moving at right angles to the direction of the star. But in any case the motion of the telescope can be resolved into two components, one along and the other perpendicular to the direction of the star. The former merely causes a virtual change in the velocity of light, and to this extent produces possibly a second-order effect. Practically the transverse component will operate as though it alone existed, and the effect to be expected may be inferred from the simpler case already considered.

It must be understood that this is a description of what appears to take place according to the evidence of observation, and not an explanation why it takes place. Otherwise we should encounter the difficulties presented by the much discussed Michelson-Morley experiment, and that is not our intention at present.

3. We turn now to the case of the water telescope investigated experimentally by Airy. Let fig. 2 take the place of fig. 1 when the tube of the telescope is filled with a medium of refractive index μ. If the light disturbance were propagated unaltered along cf it would travel with the velocity U/μ to its focus at f, and the aberrational displacement would be $Ff = \mu f V/U$. The angle fCF is thus the small angle $\tan^{-1} \mu V/U$, but this corresponds to a displacement of the *incident* ray, which may be represented with sufficient accuracy by $\tan^{-1} \mu^2 V/U$. Numerically this corresponds to a maximum aberration of about 36″. Actually, as we know, the Greenwich observers detected no change in the aberration of the star γ Draconis.

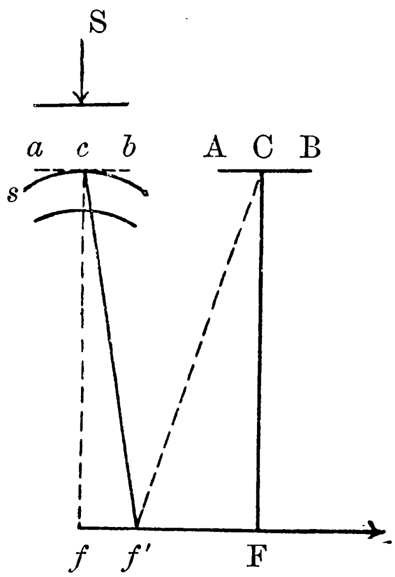


FIG. 2.

If we suppose that this is due to a lateral drift of the waves in the direction of motion, it is easy to calculate the necessary velocity v. The result is that they converge to a focus at f' where $f'F = fV/\mu U$. For this will correspond to the required apparent aberration of the incident ray. It is to be remarked here that the linear displacement $f'F$ is measured by a micrometer which has been calibrated by measuring the known angular distances between external objects, actually by turning the telescope through a right angle about its axis and observing transits of a known star. We also have $ff' = \mu f v/U$. Hence, since

$$fF = ff' + f'F,$$

$$\mu f V/U = \mu f v/U + f V/\mu U$$

or

$$v = V(1 - \mu^{-2}),$$

which is the well-known result, and accords with Fresnel's theory and the celebrated experiment of Fizeau. The physical interpretation can be left on one side.

4. These aspects of the theory are recalled, not because they contain anything new, but that we may have a clear idea of the apparent facts. The result of the experiment with the water telescope seems to show that the astronomical results are unaffected by the fact that they are made in air and not *in vacuo*. Moreover, the interpretation of the experiment leads to the enunciation of a law as to the apparent drift of the light waves in a dense medium, which may be applied to the investigation of a difficulty which we have already noticed. This difficulty is concerned with what happens to the light-disturbance between the time when it reaches the outer surface of the lens as a plane wave and the time when it has just left the inner surface as a converging spherical wave. This point has commonly been left without discussion in astronomical treatments of the question.

The problem is difficult, and it is not proposed to discuss it very deeply here. But in order to draw attention to it, some examination may be given to the simplest possible case, *i.e.* that first discussed above, but with the condition that the path of the light shall be entirely *in vacuo*. This means that the lens system must be dispensed with, and for it must be substituted a parabolic mirror to serve as the image-forming apparatus.

5. Let BAD (fig. 3) be a section of the mirror at the time t when the advancing plane wave has reached the position BD, A

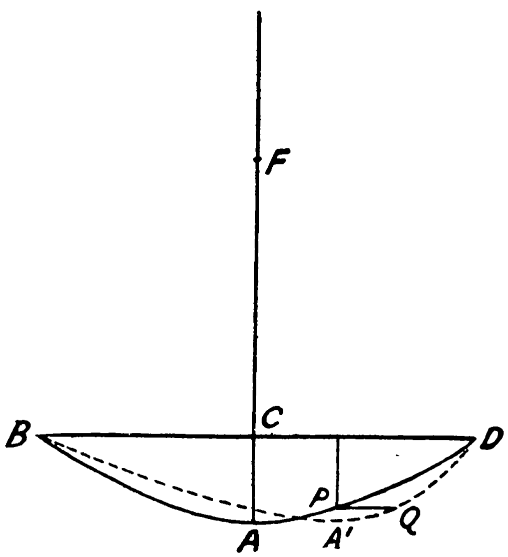


Fig. 3.

the vertex, and F the focus. Take CF as the axis of x and CD as the axis of y, these being fixed in the ether while the mirror is moving in the direction CD.

The equation of the parabola at the time t is

$$y^2 = 4f\,(x+b)$$

where $\mathrm{CA} = b$. The point P of the mirror will not receive an

element of the advancing wave at the time t, but at a subsequent instant when it has advanced to Q. If (x', y') be the point P and (x, y) the point Q we have

$$x = x', \ y - y' = -\mathrm{V}x/\mathrm{U} = -x \tan \beta,$$

so that (x, y) lies on the curve

$$(y + x \tan \beta)^2 = 4f(x + b),$$

which can also be written in the form

$$(y \cos \beta + x \sin \beta - 2f \sin \beta \cos^2 \beta)^2$$
$$= 4f \cos^3 \beta \ (x \cos \beta - y \sin \beta + b \sec \beta + f \sin^2 \beta \cos \beta).$$

We see at once that the effect of the motion is to cause the figure of the mirror to undergo a virtual deformation. The section considered is still parabolic, but the axis is no longer perpendicular to the circular sections, and the figure ceases to be a surface of revolution. The effect must be to cause a distortion of the image.

The vertex A′ of the deformed section is at the point

$$x = -b + f \sin^2 \beta \cos^2 \beta$$
$$y = b \tan \beta + f \sin \beta \cos \beta \ (1 + \cos^2 \beta),$$

while the focal length is $f \cos^3 \beta$ and the axis is turned backward through the angle β. Thus if F′ is the focus (fig. 4), MN the

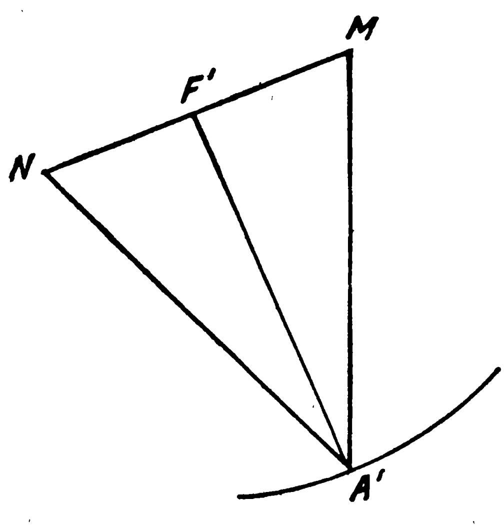


Fig. 4.

trace of the focal plane, and N the point where light coming in the direction MA′ is brought to a focus, $\mathrm{MA'F'} = \mathrm{NA'F'} = \beta$,

$A'F' = f\cos^3\beta$ and $A'N = f\cos^2\beta$. Hence the coordinates of N are

$$\begin{aligned} x &= -b + f\sin^2\beta\cos^2\beta + f\cos^2\beta\cos 2\beta \\ &= -b + f\cos^4\beta \\ y &= b\tan\beta + f\sin\beta\cos\beta\,(1+\cos^2\beta) - f\cos^2\beta\sin 2\beta \\ &= b\tan\beta + f\sin^3\beta\cos\beta. \end{aligned}$$

Now the length of the path from the wave-front BD to N *via* A′ is

$$b - f\sin^2\beta\cos^2\beta + f\cos^2\beta = b + f\cos^4\beta,$$

so that by the time the light has reached N the telescope as a whole will have moved through a distance $b\tan\beta + f\sin\beta\cos^3\beta$. Hence to the first order the light-time can be taken as $(b+f)/U$, and the displacement of the image as $f\tan\beta$, which is consistent with the law of aberration. Here we have been considering the light incident on the deformed parabolic section near its vertex. But the path from all points of the circular section BD to F is equal to BF or $b+f$, and hence the light will arrive at F at the same time in the same phase, and will form part of the image. For this part the aberrational displacement will be $(b+f)\tan\beta$. Hence we see that the result of the motion of the telescope is to produce confusion of the image, and that the consequence may be to displace the centre.

6. The actual amount of this virtual distortion of the figure of the mirror is extremely small. The maximum distance between the actual and the virtual surfaces may be expressed with sufficient accuracy by

$$\Delta x = \tan\beta . Dr^2/96\sqrt{3},$$

where D is the diameter of the mirror and r is the ratio of the aperture to the focal length. Now $\tan\beta = 10^{-4}$, $1/96\sqrt{3} = 6.10^{-3}$ and $\lambda = 2.10^{-5}$ in., the wave-length of yellow light. Hence

$$\Delta x = 0{\cdot}03\lambda r^2 . D/1 \text{ in.}$$

Thus if r is 1 : 5 and D = 60 in., the diameter of the largest mirror in being, the maximum separation of the real and the virtual surfaces is $0{\cdot}07\lambda$, a quantity just on the verge of what a practical optician will regard as sensible.

7. On the basis of the preceding investigation we may estimate the proportionate effect on the observed aberration as of the order of $+b/2f$ in the particular case considered. With a ratio of aperture to focal length of 1 : 5, the fraction becomes 1/800, which corresponds to a maximum aberrational effect of $0''{\cdot}025$ additional to the ordinary aberration. The magnitude is small, but it is precisely of the order of the discrepancy which has hitherto remained without explanation.

The question then arises whether this may not be a true explanation, and whether it is not possible that in the more complicated case of a lens the explanation of a residual $0''{\cdot}05$ in

the determination of the aberration constant may be traced to the distortion of the image which arises from the virtual deformation of the surfaces of the lens as a result of the motion through space. In this direction seems to lie a promising clue by which the apparent antagonism between two all-important astronomical constants may be evaded. But it is one which, on further consideration, must be set aside as untenable. The above investigation, it must now be confessed, is based on a faulty idea. We are in fact dealing with one of those seductive errors the close scrutiny of which has been instructive to the writer, and may possibly assist others to clearer views of a singularly elusive problem.

8. Before explaining the answer to the question to which we have been led—very unnecessarily as some may think—it may be profitable to consider another example of the application of our present point of view. Until we can find a more general principle it will be necessary in all questions relating to aberration to consider in detail the path of the light throughout the entire optical system with which we are dealing. A simple and apposite illustration suggests itself readily in the treatment of meridian observations by reflexion. The nadir distance of the reflected ray is equal to the zenith distance of the direct ray if aberration is not considered. In actual practice the nadir distance is treated exactly as if it were a zenith distance, and aberration is applied to both in the same way, as if the two were really the same thing. As a practical rule the method is correct, but the justification for it is not commonly understood. The motion of the Earth may be resolved into three rectangular components, of which we are now concerned with (1) the horizontal component towards the south, (2) the vertical component towards the nadir. The effect of these may be considered successively. Now the effect of (1) is to increase the zenith distance of the direct ray (for a south star), and equally to increase the nadir distance of the reflected ray, so that both observations are affected equally in the same sense. But it is otherwise in the case of (2). The effect of a positive component towards the nadir is to increase the zenith distance of the direct ray, but to diminish the nadir distance of the reflected ray. Hence if both observations are treated alike, we might expect a discrepancy of twice that part of the aberration which arises from the vertical component of the Earth's orbital velocity. What is the explanation of this apparent paradox?

9. It is that we have only considered the purely *apparent* effects of aberration, which depend upon the motion of the telescope. Here there is no real change in the paths of either the direct or the reflected ray. But there is also a real change in the direction of the reflected ray, due to the fact that the reflecting mercury surface is moving parallel to itself. Just as in the case of the concave mirror we were concerned with a virtual deformation of the figure owing to its motion in space, so again here we have a virtual tilt of the plane mirror which is of the same nature.

Let (fig. 5) AB be the position of the incident wave-front at the time when AC is the position of the mirror. The element of the wave at B will reach the mirror at F, where CF/BF $= v/\mathrm{U}$, v being the vertical component of the Earth's velocity. Thus the mirror

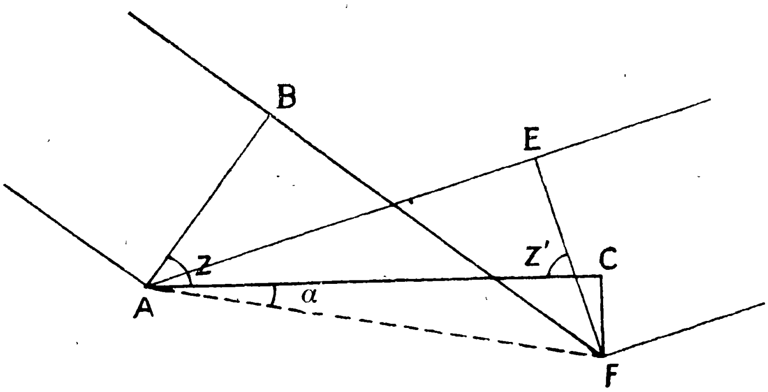


FIG. 5.

is virtually tilted into the position AF, and the light is reflected from the surface AF according to the ordinary law. Hence if FE is the reflected wave-front, AB = FE. Let Z be the angle of incidence BAC, Z′ the angle between EF and AC, and α the angle CAF. Then

$$\mathrm{Z} + \alpha = \mathrm{BAF} = \mathrm{AFE} = \mathrm{Z}' - \alpha$$

$$\alpha = \tfrac{1}{2}(\mathrm{Z}' - \mathrm{Z})$$

$$\sin \tfrac{1}{2}(\mathrm{Z}' - \mathrm{Z}) = \mathrm{CF/AF} = v.\mathrm{BF/U.AF} = v \sin \tfrac{1}{2}(\mathrm{Z} + \mathrm{Z}')/\mathrm{U}.$$

Hence

$$\tan \tfrac{1}{2}\mathrm{Z}' = \frac{\mathrm{U} + v}{\mathrm{U} - v} \tan \tfrac{1}{2}\mathrm{Z}$$

which is the law of reflexion for any velocity of the mirror, and to the first order

$$\mathrm{Z}' - \mathrm{Z} = 2v \sin \mathrm{Z}/\mathrm{U}.$$

This in the astronomical application is the increase of the nadir distance of the reflected ray owing to the vertical motion of the mercury surface, and it is precisely the quantity required to compensate the discrepancy to which our attention had been led. The above exact law of reflexion at a moving surface is a result given by Professor Hicks* in his discussion of the Michelson-Morley experiment.

10. Now the compensation here is a particular case of Veltmann's theorem,† according to which the common motion of

* *Phil. Mag.*, 1902 Jan. † *Pogg. Ann.*, cl. p. 497.

light source, optical system, and observer can produce no first-order effects in the phenomena of reflexion, refraction, or interference. The general theorem, though simple, cannot be regarded as absolutely obvious. In the meridian observations by reflexion we should find a verification of an exceedingly high order of precision were it not for the existence of the well-known R – D discordance. But the case of the concave mirror considered above seems at first sight inconsistent with the general theorem. The difficulty is, however, easily removed when we realise that it can only be an apparent one.

In the first place the virtual deformation in the shape of the mirror is not the serious matter which we may have supposed without a little closer examination in § 5. Taking a third axis at right angles to the principal section, we have for the equation of the deformed surface on which the wave-front actually falls

$$z^2 + (y + x \tan \beta)^2 = 4f(x + b).$$

This, as stated, is not a figure of revolution. But the equation of the paraboloid of revolution which has the same section in the xy-plane proves to be

$$z^2 \sec^2 \beta + (y + x \tan \beta)^2 = 4f(x + b),$$

which shows that the deviation of the former from a figure of revolution is only of the second order. And now the explanation of the confusion of the image is not far to seek. The incident ray is inclined at an angle β to the axis of the virtual surface of the mirror, and the light is not focussed at a point on the axis. Hence the confusion of the image found above is real enough, but it is due to nothing else than the aberration in the sense in which the word is used in geometrical optics. It is a natural and necessary consequence of the properties of the parabolic mirror, and it has no bearing whatever on the real question we are considering. Properly considered it is precisely the effect which ought to have been expected.

11. A very simple and general proof of Veltmann's theorem has been given by Potier.* On account of its great importance a slightly modified form of proof may be given here. We have seen in § 3 that in a medium moving with a velocity V in the direction of a fixed axis Ox the light-waves drift in the same direction with a velocity $(1 - \mu^{-2})$V. The result is as if the light-disturbance were propagated in the medium at rest while the material points in the medium are moving independently with a velocity μ^{-2}V in the given direction. Let PQ be an element ds of a path of light, making an angle ϕ with Ox, when the medium is at rest. When it is in motion the corresponding element of path will be PQ′ where QQ′ is parallel to Ox and

$$\mathrm{QQ'}/\mathrm{PQ'} = \mu^{-2}\mathrm{V}/\mu^{-1}\mathrm{U} = \mathrm{V}/\mu\mathrm{U}.$$

* *Journal de Physique*, 1st series, vol. iii. p. 201.

Hence, since

$$PQQ' = 180° - \phi,$$
$$\sin QPQ' = V \sin \phi/\mu U$$

and therefore

$$ds/PQ' = \sin PQ'Q/\sin PQQ' = \sin (\phi - QPQ')/\sin \phi$$
$$= \sqrt{(1 - V^2 \sin^2 \phi/\mu^2 U^2)} - V \cos \phi/\mu U.$$

To the first order then

$$ds' = PQ' = ds + V \cos \phi . ds/\mu U$$
$$= ds + V dx/\mu U.$$

Now the path PQ′ is travelled with a velocity $\mu^{-1}U$. Thus

$$\int \mu ds' = \int \mu ds + V(x_1 - x_0)/U,$$

which shows by Fermat's principle that the path of a ray in the medium at rest, between two points whose abscissæ are x_0 and x_1, must also be the relative path of a ray when the medium is in motion, because if one integral is a minimum the other must be a minimum also; and further, that the retardation in the time of arrival at one point in the medium from another by all rays is constant and equal to $V(x_1 - x_0)/U^2$. Hence all possible paths between two given points introduce the same difference of phase, and the motion of the medium gives rise to no first-order optical effect when source and receiver share the same motion.

12. We are now in a position to state the problem of aberration in a clear way, and to distinguish between the astronomical and the physical aspects of the question. The light leaves the source at the time T and arrives at the time *t*. Let us suppose the source at the time T to be replaced by a fictitious luminous body occupying the same position but moving with the same velocity as the observer. If now we say that the direction of the actually observed ray is the true direction of the fictitious body at the time *t*, we have three points to consider :—

(1) Whether the apparent direction of the fictitious body really is unaffected by the common motion of itself and the observer through space?

(2) Whether the substitution of the fictitious for the true source of light is legitimate. In other words, is the direction of the received ray independent of the motion of the source in space?

(3) That the direction of the actual source of light can be inferred from that of the fictitious source.

The first two of these points are physical questions. The third is purely a statement of kinematical fact. The three together appear to cover the whole problem, excluding considerations which are foreign, and including what is really germane to the theory.

13. We see at once that by assuming a favourable answer to the physical questions we are left with a simple geometrical problem, the solution of which is effected by the ordinary astronomical rules. The different forms which these rules assume when they are adapted to deal most conveniently with different circumstances have been very fully examined by Professor Turner (p. 403), who has brought out their essential harmony by the detailed discussion of several illuminating examples. From our present point of view the astronomical effect of aberration depends solely on the light-time from the source at time T to the observer at time t.

This means that we are concerned with the actual wave-velocity of light in the medium through which the light travels. If space is filled with a dispersive medium the light-time will be to some extent affected. Recent investigations have suggested that such a medium may exist. But it cannot in any case be competent to exercise any practical influence on observed aberration. For if the light-time were increased in this way by one part in 10,000 the effect on the aberration could not be detected, while, on the other hand, a medium capable of producing an effect of this magnitude would be so strongly absorptive as to render all astronomical observations impossible. The Earth's atmosphere exercises no appreciable effect in this connection, because the light-time through it is merely a very small fraction of a second.

14. Here it may be remarked that the velocity of light does not appear to have been determined by experiment with the extreme certainty which is commonly supposed. By the rotating-wheel method Cornu obtained

$$300400 \text{ km./sec.},$$

while by the revolving-mirror method Newcomb obtained

$$299860 \text{ km./sec.},$$

in close agreement with Michelson's earlier result by the same method. The discrepancy, which is of the order of one part in 500, has been discussed very carefully by Cornu,* who concluded that his result was worthy of no less confidence than the other, in spite of its greater probable error. Rosa and Dorsey's recent very accurate determination of the electromagnetic velocity

$$299710 \text{ km./sec.}$$

may be set down here, although it is only made comparable with the other numbers by theory, and it involves a slight uncertainty in the absolute value of the international ohm.

There can be little doubt that the solar parallax is now known certainly by direct determination within one part in 1000. On the

* *Rapports du Congrès de Physique à Paris*, 1900, vol. ii. p. 225.

other hand, the direct determination of the constant of aberration has left the value uncertain by scarcely less than one part in 400. This is unfortunate, and it is of fundamental importance to discover the cause of this uncertainty. Meanwhile the constant of aberration has been determined by the application of the Doppler principle. The detailed account of this work at the Cape Observatory has not yet been distributed, but it is understood that the combination of this result with the accepted value of the solar parallax yields a value for the velocity of light which is in close agreement with Newcomb's experimental result.

15. We may now consider briefly the two physical questions raised in § 12. To the first order, as Veltmann's theorem shows on the simplest principles of wave theory, the apparent direction of a body which shares with the observer a common motion through space is unaffected. Curiously enough, it is this absence of effect which is of the greatest importance to physical theory. But it does not require an astronomical observation for its verification. It is true that the scale of the experiment is immense when it is extended to the furthest stars, and to some minds this may suggest that we are here almost on the verge of a metaphysical question respecting the character of the straight line in space. But in a practical sense the cosmic scale of the apparatus is scarcely relevant. The point can be tested quite as well in the laboratory, and here a far subtler verification can be devised than is to be obtained from the estimated position of a star-image in the field of a telescope, namely, by a well-designed interference experiment. A very wide range of delicate experiments of this kind was executed by Mascart, but the very highest accuracy was reached in the Michelson-Morley experiment, which has been repeated in recent years with increased refinement by Morley. Unfortunately for the theory, not only is the first-order effect compensated, but the expected second-order effect is also absent, although the sensitiveness of the test is adequate to have shown it if it had existed. This null result has led to developments in modern physical theory which are beside our present purpose. The absence of the first-order effect is what directly concerns us. This at least seems certain; and thus it appears impossible that any astronomical observation can be sensibly influenced by the nature of the telescope in relation to its motion through space. The question raised in the early part of this paper is thus answered decidedly in the negative.

The second physical question is concerned with the nature of radiation. We have to admit that the way in which the radiation from a source of light is propagated is quite independent of the motion of the source in space. This is certainly true on the wave theory, and it may be difficult to conceive the contrary. But it appears to be a condition which requires statement.

16. This paper has gone further than was at first intended without throwing any light on the causes of the discrepancies which have been found in the determination of the constant of aberration. But it will have served its purpose if it has rendered

the nature of the problem a little clearer, and if it has helped to close any of those paths which, however inviting, can lead to no solution. In conclusion, the paper may be summarised thus:—

§ 1.	Introductory.
§ 2.	Elementary explanation of aberration *in vacuo*.
§§ 3, 4.	Theory of Airy's water telescope.
§§ 5–7.	Theory of aberration when the image is formed by a mirror.
§§ 8, 9.	Theory of aberration in the case of meridian observations by reflexion.
§ 10.	Reconciliation of the result of §§ 5–7 with Veltmann's theorem.
§ 11.	Proof of Veltmann's theorem.
§ 12.	Distinction between the astronomical and physical principles involved.
§§ 13, 14.	Function and values of the velocity of light.
§ 15.	Consideration of the physical conditions.

Spectroscopic Comparison of o Ceti with Titanium Oxide.
By A. Fowler. (Plate 19.)

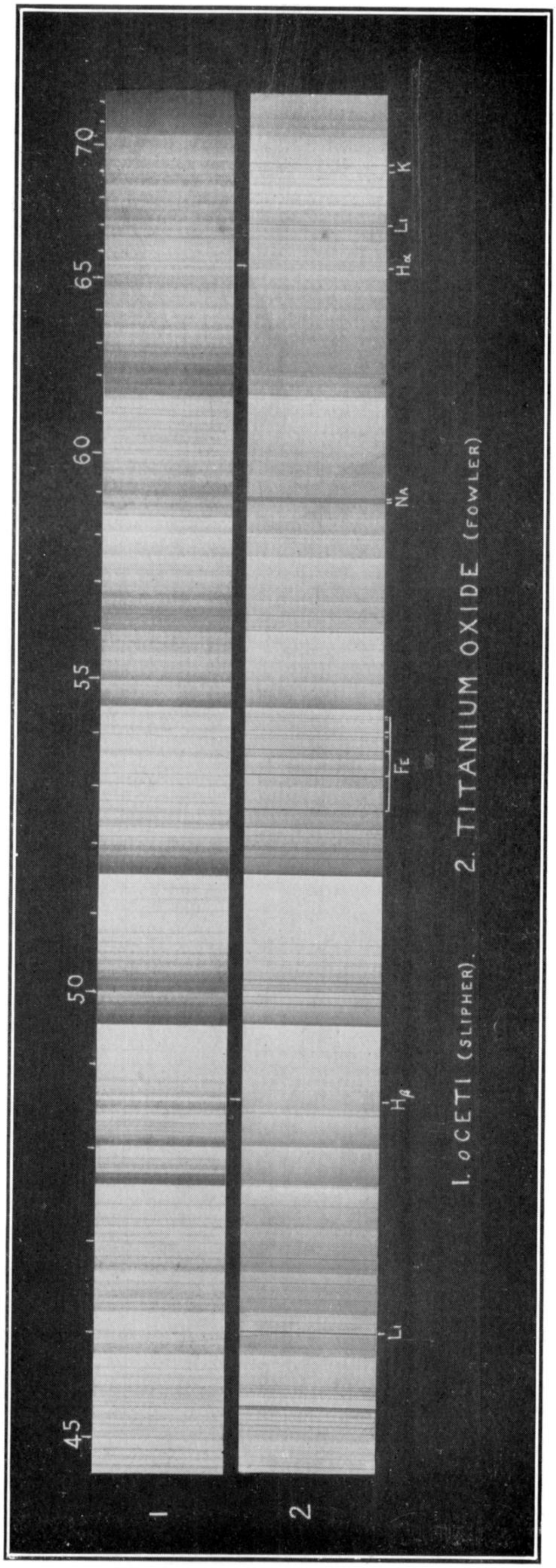
It has already been shown that nearly all of the characteristic flutings of the Antarian or Third Type Stars are produced by the absorption of titanium oxide.* The chief purpose of the present note is to give an additional comparison which strikingly illustrates the truth of the identification.

Having no adequate instruments for the photography of stellar spectra, I appealed to Mr. Slipher, of the Lowell Observatory, for permission to utilise his excellent photograph of the spectrum of *o* Ceti, which is admirably adapted for the purpose in view because it extends from the extreme visible red to the violet.† Mr. Slipher not only consented to this use of the photograph, but very kindly forwarded a special copy for reproduction. The resulting comparison with the fluted spectrum of titanium oxide is shown in Plate 19, in which the stellar spectrum is about five times the scale of the original negative.

The titanium oxide spectrum was photographed on one of Wratten's panchromatic plates with a Littrow spectrograph, giving a dispersion not too great for appropriate comparison with the star. It was obtained from the flame of the electric arc when a considerable quantity of the substance was volatilised between iron poles, and shows a few lines of iron and titanium in addition to the flutings. The D lines of sodium, the red and blue lithium lines at 6708 and 4602, a group of potassium lines about 5800, and the red

* *Proc. Roy. Soc.*, vol. 73, p. 219 (1904). *Monthly Notices* (reprint), App., vol. lxiv. p. 16 (1904). *Proc. Roy. Soc.*, vol. 79, p. 509 (1907).

† *Astrophys. Jour.*, vol. 25, p. 236 (1907).



SPECTROSCOPIC COMPARISON OF *o* CETI WITH TITANIUM OXIDE.